



# Electronic-Structure Solvers in SIESTA: Features and Performance

Alberto García (ICMAB-CSIC, Barcelona)



# The basic core of SIESTA

$$\psi_i(\mathbf{r}) = \sum_{\mu} \phi_{\mu}(\mathbf{r}) c_{\mu i},$$

$$\sum_{\nu\beta} (H_{\mu\nu}^{\alpha\beta} - E_i S_{\mu\nu} \delta^{\alpha\beta}) c_{\nu i}^{\beta} = 0$$

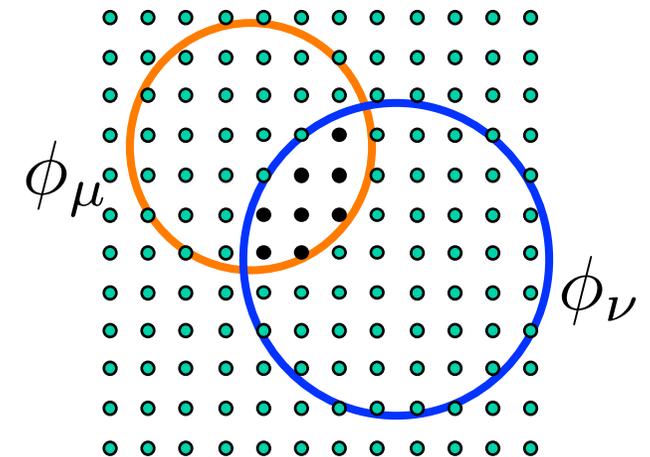
Generalized eigenvalue problem

$$\rho(\mathbf{r}) = \sum_{\mu\nu} \rho_{\mu\nu} \phi_{\nu}^*(\mathbf{r}) \phi_{\mu}(\mathbf{r})$$

$$S_{\mu\nu} = \langle \phi_{\mu} | \phi_{\nu} \rangle$$

$$\rho_{\mu\nu} = \sum_i c_{\mu i} n_i c_{\nu i}$$

Density matrix



The SOLVER step takes most of the CPU time

# Diagonalization-based solvers

Need to use DIRECT solvers, as the number of desired eigenvectors is a substantial fraction of the matrix size

$$\sum_{\nu\beta} (H_{\mu\nu}^{\alpha\beta} - E_i S_{\mu\nu} \delta^{\alpha\beta}) c_{\nu i}^{\beta} = 0$$

SIESTA uses pre-packaged libraries for this pure math problem:

- **ScaLaPACK**
  - pdsyev, pzheev and related drivers
  - MRRR
- **ELPA**: Alternative transformation sequence + optimizations  
<https://elpa.mpcdf.mpg.de/>

- **Conversion of H and S to dense form**
- Cholesky decomposition to reduce to standard eigenproblem
- Transformation to tri-diagonal form
- Solution of tri-diagonal problem
- Back-transformation

**Cubic scaling** with matrix size — Quadratic scaling for memory

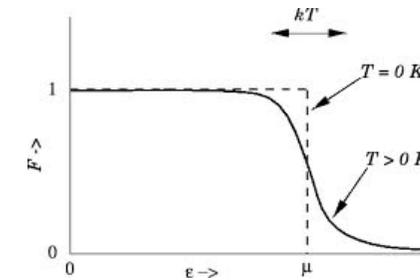
Still competitive for low-cardinality basis sets

# Direct solution for the density matrix

$$\hat{\rho} = f_{\beta}(\hat{H} - \mu)$$

$$f_{\beta}(\epsilon_i - \mu) = \frac{2}{1 + e^{\beta(\epsilon_i - \mu)}}$$

## Fermi-Dirac function



## Fermi Operator Expansion (FOE)

$$p(H) = \frac{c_0}{2} I + \sum_{j=1}^{n_{pl}} c_j T_j(H)$$

Calculation of the DM involves only (sparse) matrix-vector multiplications

CheSS library

(originally in BigDFT)

Linear-scaling



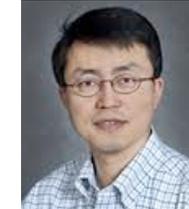
Stephan Mohr (BSC)

- Number of terms in the expansion can be large
- Efficiency increases for contracted basis sets.
- Exploring on-the-fly contraction

# Direct solution for the density matrix

## PEXSI: Pole Expansion plus **Selected Inversion**

(Lin Lin, Chao Yang, et al., Berkeley)

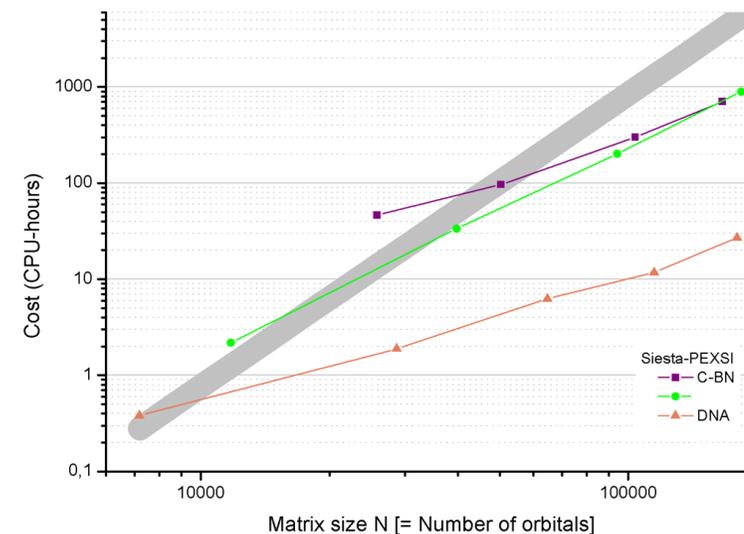


$$\hat{\rho} = \text{Im} \left( \sum_{l=1}^P \frac{\omega_l}{H - (z_l + \mu)S} \right)$$

For sufficiently big problems  
(quasi-)1D:  $\mathcal{O}(N)$   
(quasi-)2D:  $\mathcal{O}(N^{3/2})$   
3D:  $\mathcal{O}(N^2)$

(Due to sparsity of the target density matrix)

Relatively small number of poles (20-30)  
Trivially parallelizable over them

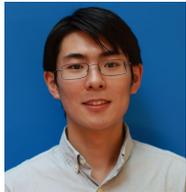


# Solver strategies for performance and features: Use external libraries

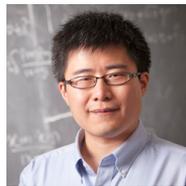
## ELSI initiative to integrate solver libraries



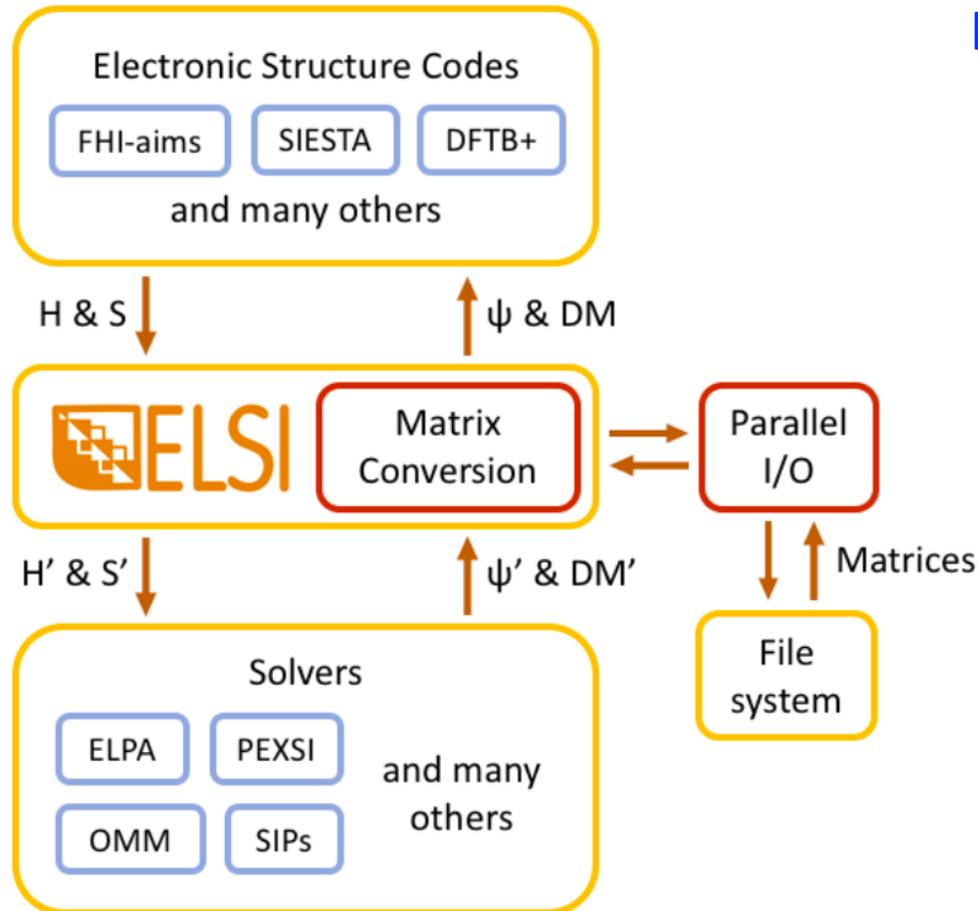
Volker Blum, Duke



Lin Lin, Berkeley



Jiangfen Lu, Duke



<https://elsi-interchange.org>

Interface in Siesta:

Collaboration with  
Victor Yu (Duke)



# Solver strategies for performance and features: Use external libraries

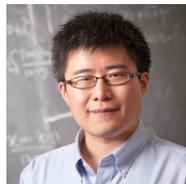
## ELSI initiative to integrate solver libraries



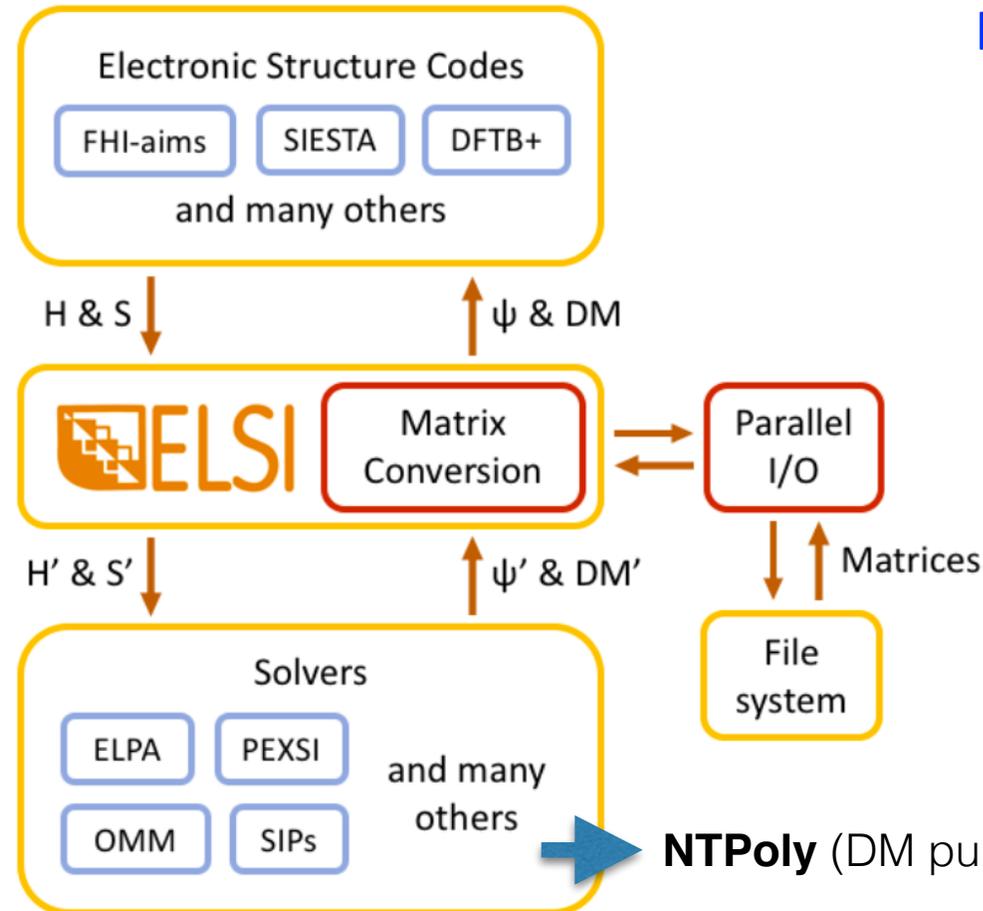
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Interface in Siesta:

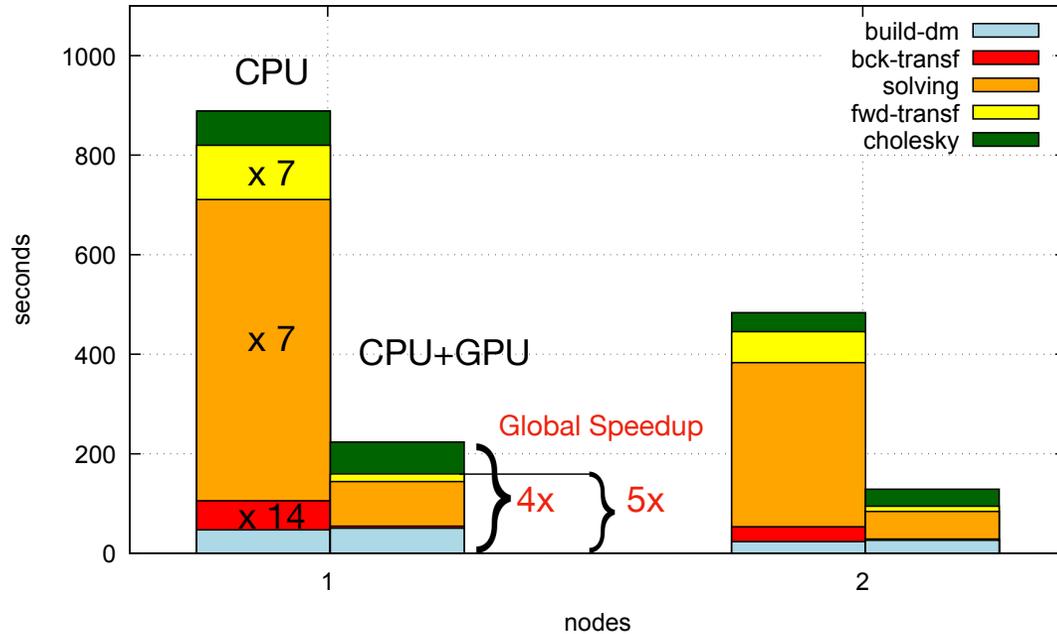
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# GPU acceleration for diagonalization

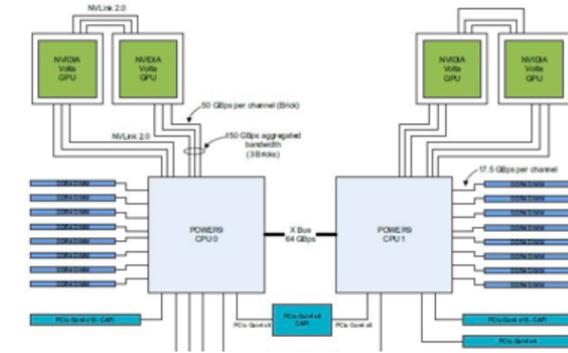
Marconi-100 (CINECA): 32 CPUs+ 4 GPUs /node

ELSI-ELPA GPU acceleration

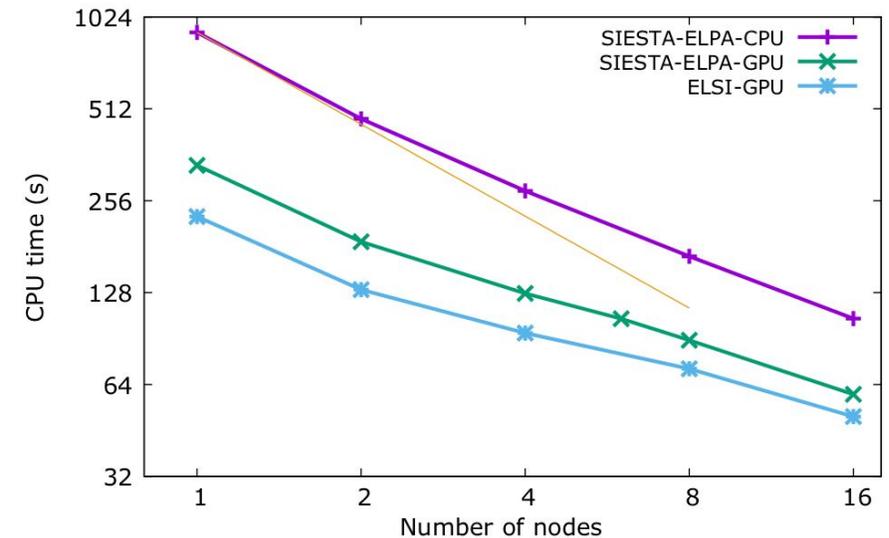


Future enhancements in ELPA (better kernels) and in ELSI (e.g. build-DM stage) are integrated in SIESTA automatically

System: Si quantum dot, with approx 35000 orbs



Proper binding of GPUs to MPI ranks

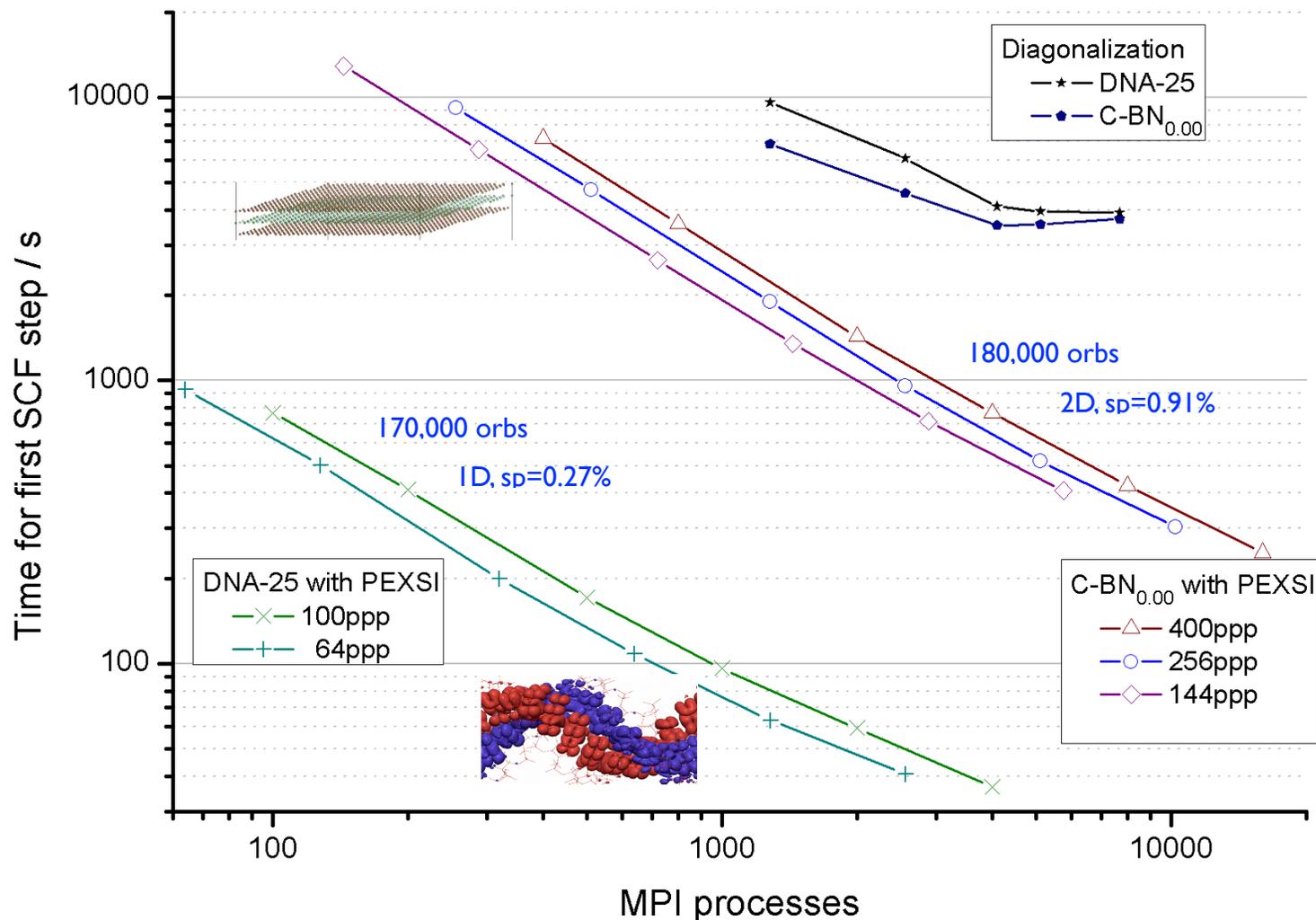


# Massive scalability: PEXSI solver

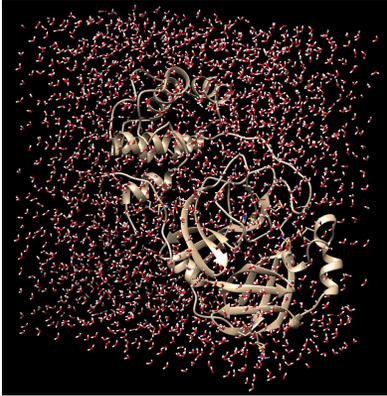
$$\hat{\rho} = \text{Im} \left( \sum_{l=1}^P \frac{\omega_l}{H - (z_l + \mu)S} \right)$$

PEXSI offers:

- Three levels of parallelization (over orbitals, poles, and chemical potential values)
- A reduced memory footprint (only sparse matrices are stored)
- Reduced complexity (maximum  $O(N^2)$  size scaling)

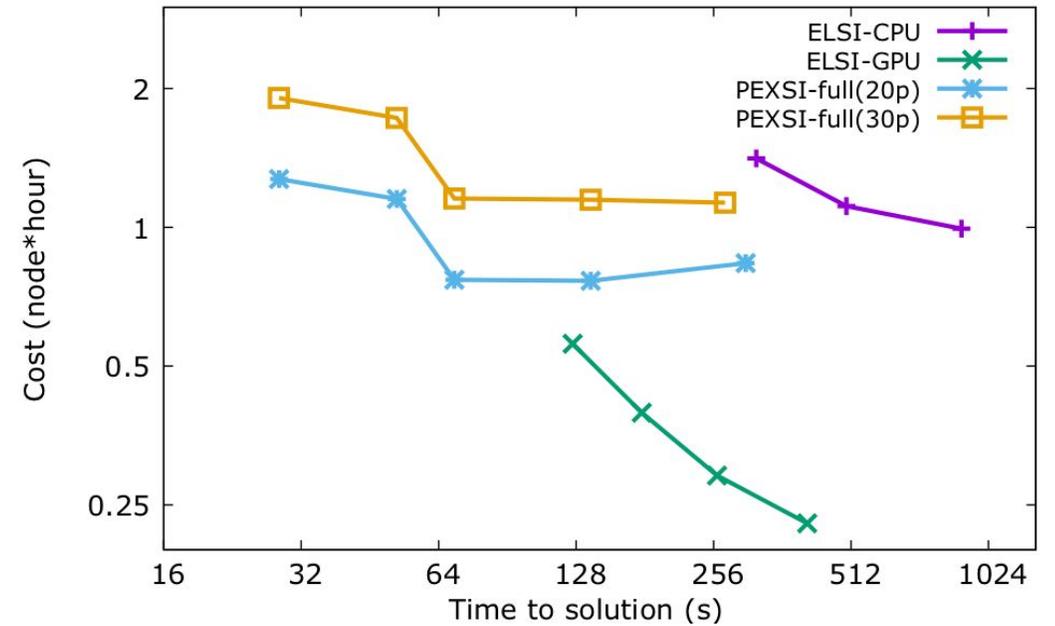
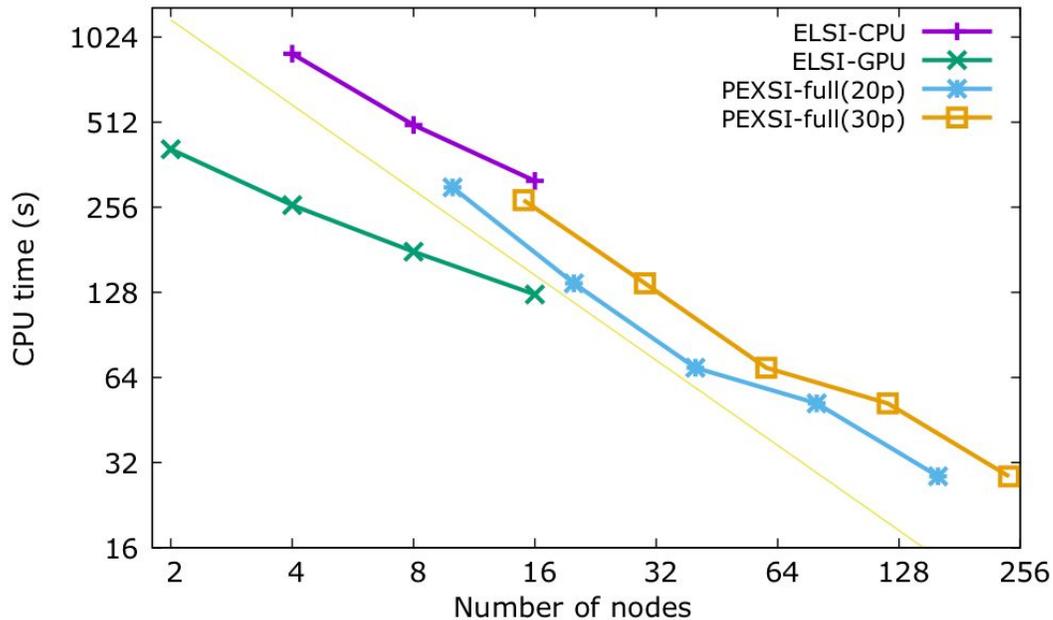


# Comparison of global efficiency of solvers for a very large problem



SARS CoV-2 M<sup>pro</sup> with solvation water molecules

Approx 8800 atoms; 58000 orbitals



Work on GPU acceleration of PEXSI library is under way



DRIVING THE EXASCALE TRANSITION

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THANKS